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Life Cycle Management of Structural Systems Based on the Optimal SHM Strategy by VoI Analysis

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ABSTRACT: Structural health monitoring (SHM) has been widely installed on critical infrastructure, such as buildings, bridges, dams, etc., which are very beneficial for optimal life-cycle decision making. However, the identification of how to implement SHM optimally on a structural system is a key challenge in Structural Integrity Management (SIM). In this paper, the theory of Value of Information (VoI) is applied to make the optimal SHM strategy decision for deteriorated structural systems in the context of life cycle management. The VoI is quantified by the difference of life-cycle cost between the prior decision analysis and pre-posterior decision analysis, with the consideration of different system properties. Taking the series systems and parallel systems as two common system models, the general performance deterioration model and routine maintenance strategy for structural components are considered. Based on the VoI analysis, the effects of different system properties on VoI are demonstrated and the optimal life-cycle SHM strategies for different structural system models are determined. Correspondingly, the prior and pre-posterior life-cycle cost of structural systems are analyzed and the related parametric analysis results show that system properties have a significant influence on the VoI.

1. INTRODUCTION

Structural Health Monitoring (SHM) has made remarkable achievements and has been widely used in large and complex infrastructure projects, such as buildings, bridges, dams, offshore platforms, power plants, etc. Information from SHM can improve the management decisions of structures, and reduce the loss of lives and damage of properties. Although the ability of SHM in evaluating the conditions of engineering structures has been widely recognized, the question on how to measure the potential benefits of SHM in practical applications has been studied only in more recent years, see Pozzi and Kiureghian (2011); Faber and Thöns (2013), Konakli et al. (2016); Zonta et al. (2013); Straub et al. (2017), among others. To effectively quantify the benefits of SHM in the life-cycle management of critical infrastructure, the Bayesian pre-posterior decision analysis introduced by Raiffa and Schlaifer (1961) is a good way to achieve that. Specifically, the concept of Value of Information (VoI) can be particularly useful in identifying efficient ways to improve the prospective outcomes for the chosen course of actions.

Faber and his colleagues firstly applied the VoI concept in the field of risk-based inspection (RBI), see Faber (2002); Straub and Faber (2005). The explicit illustrations of VoI analysis method were also studied by Straub (2014) and Konakli et al. (2016). An application of VoI for quantifying the benefit of forecasting the environment change in the risk management was studied by Roldsgaard et al. (2015). As for structural life-cycle integrity management, Thöns et al. (2015) quantified the value of SHM information in the field of fatigue deterioration, while Qin et al. (2015) made a decision of the optimal SHM operation period based on the VoI concept on the component levels. Faber (2017) and Thöns (2018) demonstrated how SHM and inspection strategies for structural risk and integrity management can be performed with the help of VoI.

This paper aims to investigate the optimization of SHM implementation strategy for deteriorated structural systems in the context of life-cycle management via the concept of VoI. First, a framework of VoI analysis in the context of life-cycle management is presented, then the life-cycle performance of structural components is modeled through a time-dependent ultimate limit state (ULS) function, in which the general performance deterioration model as well as the routine maintenance strategy and related cost models are considered. The life-cycle performance of structural components is predicated and updated by Bayesian theory using the information from SHM. The life-cycle optimal strategy for SHM implementation of structural systems based on the VoI analysis is outlined. Finally, a case study is presented to illustrate the goal of this study.

2. VALUE OF INFORMATION ANALYSIS IN THE CONTEXT OF LIFE-CYCLE MANAGEMENT

A structure can encounter multiple conditions during its service cycle, resulting in different consequences. Generally, the activities of life-cycle integrity management for structures can be divided into different kinds, e.g. inspection, monitoring, maintenance and repair (IMMR), see Faber (2017) and Moan (2018). Minimization of the expected life-cycle cost is the most widely accepted principle for life-cycle integrity management of infrastructure, while the Bayesian statistical decision theory is a good choice for optimization decisions under uncertainty, which generally includes three scenarios: prior decision, posterior decision and pre-posterior decision.

Prior decision scenario can be illustrated by the decision tree shown in Figure 1. It is assumed that just one inspection is undertaken during the life cycle, which is represented by T_s. The choice of whether to repair or not depends on the results of inspections at the time t_j. Actually, the interval between inspection and decision on repair can be any duration. For convenience and without loss of generality, it is assumed that repair decisions are executed immediately after an inspection result in the following discussions.

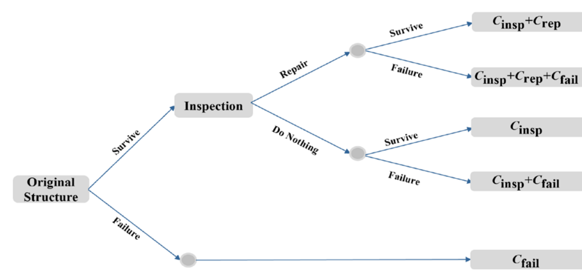

Figure 1: Prior decision analysis scenario without SHM.

In Figure 1, C_{rep}, C_{insp} and C_{fail} represent the cost of repair, inspection and failure respectively. The life-cycle cost model for an engineered structure can be written as a function of the inspection time t_j:

$$C_{SL}(t_j)=C_{fail}(t_j)+C_{insp}(t_j)+C_{rep}(t_j)+C_{rep,F}(t_j)+C_{nrep,F}(t_j) \quad (1)$$

where, $C_{rep,F}$ is the costs of repair at time t_j and subsequent failure, and $C_{nrep,F}$ is the costs of no repair and subsequent failures. The cost components C_{insp}, C_{rep} and C_{fail} associated with inspections, repairs and failures are set to 1, 10 and 1000 respectively and assumed to be constant over time.

To improve the knowledge on structures, SHM is adopted to continuously monitor the deterioration of structures before decisions. Decision makers generally optimize decisions to decrease the life-cycle cost via the SHM information. The SHM information can be divided into two types: perfect information and imperfect information. The former refers to the information collected without any uncertainty associated with the state of degradation. Obviously, this type of information is almost impossible to achieve in practice; however, it can give the upper bound of potential benefits to be achieved through monitoring. The latter can be modelled in many forms, including: inequality information; equality information, see Straub (2014); and regarding it as a sample from the probability model, see Qin et al. (2015).

The posterior decision scenario is illustrated in Figure 2. The optimal implementation strategy will be discussed in Section 5. The difference between the posterior decision analysis with specific information, which can be regard as a realization of the prior PDF, $\boldsymbol{X}=\boldsymbol{x}$, and the prior expected value of life-cycle benefits, is denoted as the conditional value of information (CVI), i.e. $\mathrm{CVI}(\boldsymbol{x})$:

$$\mathrm{CVI}(\boldsymbol{x}) = C(\boldsymbol{x}, a_{\mathrm{opt}}) - C'(\boldsymbol{x}, a_{\mathrm{opt}}^{*}) \qquad (2)$$

where $C'(\boldsymbol{x}, a_{\mathrm{opt}}^{*})$ represents the posterior cost, a_{opt} is the optimal action in prior decision, a_{opt}^{*} is the conditional optimal action with the observed information $\boldsymbol{x}$.

In reality, decision makers will make decisions on monitoring before obtaining the real condition data. Such decisions can be supported by the Bayesian pre-posterior decision analysis. The purpose of pre-posterior analysis is to identify the experimental designs or information collection strategies to minimize the life cycle costs. Accordingly, the information is represented in the pre-posterior analysis through the random vector $\boldsymbol{X}$. The potential benefits are represented by the expected value of information (EVI):

$$\mathrm{EVI} = \mathrm{E}_{\boldsymbol{X}}(\mathrm{CVI}(\boldsymbol{X})) \qquad (3)$$

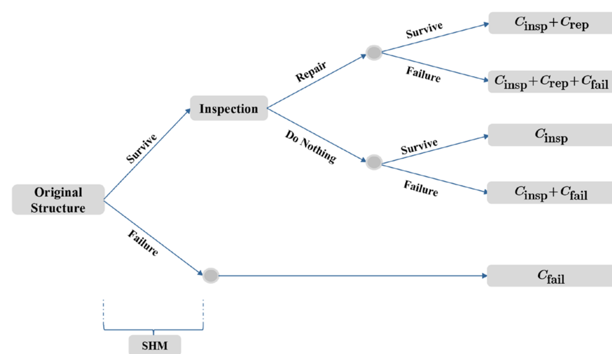

Figure 2: Posterior decision analysis scenario with SHM.

3. PROBABILISITIC MODELING OF STRUCTURAL LIFE-CYCLE PERFORMANCE AND RELATED COSTS

Probabilistic modelling of structural system performance is a crucial step in quantifying the probability of various events during the service cycle.

Structural systems are often modelled as parallel or series systems to facilitate reliability analysis. In the following discussions, the considered systems comprise 3 components with similar structural properties.

It is assumed that the condition and the failure events of the individual components are correlated, and modelled through a multivariate joint distribution with correlation coefficient ρ. The resistances and the loads in the systems are modelled with the same PDFs. The failure probabilities can be obtained by:

$$P_f = P\left(\bigcup_{i=1}^{N}\{g_i(\boldsymbol{X}) \le 0\}\right) \qquad (4)$$

$$P_f = P\left(\bigcap_{i=1}^{N}\{g_i(\boldsymbol{X}) \le 0\}\right) \qquad (5)$$

The life-cycle performance of structural components can be represented by a time-dependent ultimate limit state function

$$\mathrm{g}(\boldsymbol{X}, t) = R_0\theta_{\mathrm{d}}\left(1 - D(t)\right) - z\theta_{\mathrm{s}}S_t \qquad (6)$$

where, R_0 is the original resistance, $D(t)$ is the deterioration function, t is the time measured in number of years, z is the design parameter, θ_{d} and θ_{s} are the model uncertainty variables for the resistance and the loading respectively.

The time-varying loading process is represented by a vector of random variables representing the annual extreme loads. The deterioration function can be seen as a general degradation process:

$$D(t) = \sum_{i=1}^{t} \Delta_{\mathrm{D},i} \tag{7}$$

where the annual increments $\Delta_{\mathrm{D},i}$ follow the same distribution with the uncertain expected value M_{μ_D} and the constant standard deviation σ_{μ_D}.

To insure adequate levels of safety, it is assumed that repair will be undertaken at the time of an inspection when the degradation exceeds a threshold D_{IR}. The event of repair at the time of inspection may thus be written as:

$$\mathrm{IR}_{t_j} = D(t_j) \geq D_{\mathrm{IR}} \tag{8}$$

There are totally five different events during the life span: the event of failure F_{t_i}, the event of survival S_{t_i}, the event of inspection and repair at year t_j, IR_{t_j}, the event of failure at year t_i after inspection and repair at year t_j, $\mathrm{F}_{t_i,\mathrm{IR}_{t_j}}$, and the event of failure after no repair $\mathrm{F}_{t_i,\mathrm{nIR}_{t_j}}$. More details for illustration of these events can be found in Qin et al. (2015).

To illustrate the method in a simplified way, it is assumed that all components of structures will be repaired in accordance with the following events:

$$\mathrm{IR}_{\mathrm{series}} = \bigcup_i \mathrm{IR}_i \tag{9}$$

$$\mathrm{IR}_{\mathrm{parallel}} = \bigcap_i \mathrm{IR}_i \tag{10}$$

where i is the number of components in a system.

An analogous method is adopted to model the event of failure at year t_i after inspection and repair at year t_j and no repair at year t_j represented by $\mathrm{F}_{t_i,\mathrm{IR}_{t_j}}$ and $\mathrm{F}_{t_i,\mathrm{nIR}_{t_j}}$, the corresponding equations can be written in similar forms.

The cost model for structural systems is similar to the models for the individual components. The cost of inspection and repair for a system is equal to the sum of costs for its all components. To account for robustness in the modelling, in accordance with Baker et al. (2008), the failure cost of a system is modelled to be 1,000 times the failure costs of its individual component, see Konakli et al. (2016) and Fischer et al. (2019). In this manner, the cost models for the events with disproportional relationships between component failures and system failure can be represented as:

$$\begin{gathered} C_{\mathrm{insp}}^{\mathrm{sys}} = N \times C_{\mathrm{insp}} \quad C_{\mathrm{rep}}^{\mathrm{sys}} = N \times C_{\mathrm{rep}} \\ C_{\mathrm{fail}}^{\mathrm{sys}} = 1000 \times C_{\mathrm{fail}} \end{gathered} \tag{11}$$

4. PREDICTION AND UPDATING BY SHM INFORMATION IN VOI ANALYSIS

Information from SHM can facilitate a more efficient decision through adaptation of decisions in accordance with the collected information. The information collected from SHM for structural systems have two ways to gain benefits: prediction by conditional distribution, and updating by Bayesian statistics.

4.1. Prediction by Conditional Distribution

When the information of SHM from the monitored components is collected, denoted by $\boldsymbol{X}_{\mathrm{condi}}$, the probabilistic characteristics of the remaining components can be modelled and the probabilities associated with the states of these components can be updated by the conditional distributions. Therefore, corresponding events for non-monitored components can be represented by:

$$\left(M_{\mu_{D1}} \mid M_{\mu_{D2}} = \boldsymbol{x}\right) \sim N(M_{\mu_{D1}|\boldsymbol{x}}, \Sigma_{11|\boldsymbol{x}}) \tag{12}$$

where,

$$\begin{aligned} M_{\mu_{D1}|\boldsymbol{x}} &= M_{\mu_{D1}} + \Sigma_{12}\Sigma_{22}^{-1}(\boldsymbol{x} - M_{\mu_{D2}}) \\ \Sigma_{11|\boldsymbol{x}} &= \Sigma_{11} - \Sigma_{12}\Sigma_{22}^{-1}\Sigma_{21} \end{aligned} \tag{13}$$

in which the conditional distribution for non-monitored components is denoted by $p'_{\mathrm{condi}}(\boldsymbol{x})$ to distinguish it from the posterior distribution achieved by Bayesian updating.

4.2. Updating by Different Types of Additional Information

In the case where collection of perfect information is considered, the information can directly replace the prior distribution by the specific value which has been observed. As for imperfect information, the additional information has the potential to reduce the uncertainty through Bayesian updating. A convenient instrument for this is to take benefit of natural conjugate distributions, see Raiffa and Schlaifer (1961).

Owing to the SHM information properties, the collected information cannot reflect the true condition of the monitored object perfectly. Hence a better way might be to utilize the annual observations $\hat{\Delta}_D$ to update the uncertain hyper-parameter M_{μ_D} in the probabilistic model of the deterioration via the natural conjugate distribution. The use of natural conjugate priors is convenient since it facilitates the use of closed form solutions for establishing the posteriors, more details for updating by imperfect information can be found in Qin et al. (2015). The posterior PDF for the condition characteristics of monitored components can be obtained from the observed information. Here the updated PDF is denoted by $p'_{\mathrm{update}}(\boldsymbol{x})$.

In this paper, due to the large calculation cost of Bayesian updating by imperfect information for structural systems, only the perfect information is adopted, thus the posterior PDF will be replaced by deterministic vector $\boldsymbol{x}$.

5. OPTIMIZATION OF SHM IMPLEMENTATION STRATEGY

SHM is adopted to observe the possible relevant condition states of structural components in this paper. Possible choices regarding the implementation of SHM are the starting time for monitoring and the number of monitoring components. In the decision process, the inspection should follow the SHM process, see also Figure 2. The optimization of SHM can be undertaken according to the optimal EVI:

$$\max_{t_{\mathrm{mon,st}},n} \mathrm{EVI} = \max_{t_{\mathrm{mon,st}},n}[\mathrm{E}_{\boldsymbol{X}}(\mathrm{CVI}(p'_{\mathrm{condi}}(\boldsymbol{x}),p'_{\mathrm{update}}(\boldsymbol{x})))] \tag{14}$$

Notice that the decision regarding the initiation time for SHM might cause a negative effect on the life-cycle management when SHM is initiated too late.

As for modelling the SHM information properties, the numerical investigations performed by Straub and Faber (2003) show the influence of the inter-dependency between the properties of individual inspections on the updated system reliability is low, when the reliability of the components is moderate to high. Therefore, the consideration of the dependency between the monitoring results is neglected in this paper.

From the viewpoint of VoI, the value of more information is greater than or equal to the one associated with less information. Hence without accounting for the costs of installing and operating the SHM, it may be realized that it is more optimal to set the monitoring starting time $t_{\mathrm{mon,st}}$ at the beginning of the service cycle. Now the objective for the optimization of the SHM strategy is to identify the optimal number of monitoring components. The corresponding optimization model can be undertaken according to the optimal EVI:

$$\max_{n} \mathrm{EVI} = \max_{n}[\mathrm{E}_{\boldsymbol{X}}(\mathrm{CVI}(p'_{\mathrm{condi}}(\boldsymbol{x}),p'_{\mathrm{update}}(\boldsymbol{x})))] \tag{15}$$

6. ILLUSTRTIVE EXAMPLE

To illustrate how SHM can supply potential benefits in the context of life cycle management of structures, the series and parallel systems are formulated based on the model proposed in Qin et al. (2015). The structural system consists of 3 components with dependence performance. It is assumed that the system has a 50-year service life and has one chance to apply inspection and decision making of repair. The related probabilistic characteristics of the random variables presented in the proposed approach are provided in Table 1.

Before the implementation, the potential benefits from SHM should be used to define the strategy, therefore, The VoI analysis can be adopted. It is assumed that there is correlation of both degradation and failure event of components, described through Pearson correlation coefficient. The degradation conditions of some components can be obtained from SHM, and it can facilitate in updating the probabilistic assessment of the degradation and other components without SHM by prediction by conditional distribution.

Table 1: Probabilistic characteristics of random variables.

Variable	*Distribution*	*Mean*	*Standard Deviation*
R_0	*Lognormal*	*1*	*0.1*
θ_D	*Lognormal*	*1*	*0.1*
θ_S	*Lognormal*	*1*	*0.1*
S_i	*Gumbel*	*1*	*0.3*
Δ_D	*Normal*	M_{μ_D}	*0.1*
M_{μ_D}	*Normal*	*0.01*	*0.01*

In the investigations herein, the failure of structural components is assumed to have same correlation coefficients with each other. In this case, the life-cycle cost in the prior decision is evaluated with 10^5 Monte Carlo simulations. For the series system, the minimized expected costs, as illustrated in Figure 3, are 60567, 58631, 53389 and 44562 for the system with correlation coefficient varying from 0.2 to 0.8 respectively. For the parallel system, the life-cycle costs, as illustrated in Figure 3, are lower due to the redundancy. The minimized expected costs are 29.184, 234.38, 1121.1 and 4034.1 respectively for such four correlation coefficients. It can be seen that the best time of inspection is around the 12th year. Figures 3 indicates that the correlation of degradation has a positive impact on the expected life-cycle cost for the series system, whereas a negative influence on the parallel system.

Now consider pre-posterior life-cycle cost analysis, 10^3 posterior samples are adopted herein to calculate the value of SHM information. For illustration purpose, the SHM is installed at the beginning of service life and works till the decision point to collect degradation information. The related results can be seen in Figures 4 and 5.

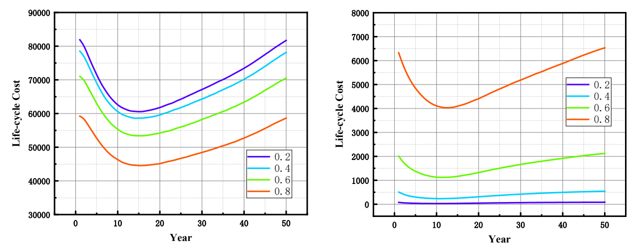

Figure 3: Expected life-cycle cost with various correlation coefficients for series (left) and parallel (right) system without SHM.

For the series system, the value of information of annual deterioration is decreased with the increase of the correlation coefficient, while the increase of the potential benefits by the additional information is not significant when the degradation of components has strong dependency. Therefore, if the investment to install one more monitoring channels and operation costs is sufficiently high, then the choice of adopting more monitored components might not be an economic decision. Opposite to the series system, the correlation has the reverse influence on VoI for the parallel system. Moreover, with the increase of correlation, more information cannot give equally increased values.

Because of the existence of the large difference in prior costs of the systems, the comparison of the absolute VoI cannot reflect the influence of SHM clearly. Therefore, the behavior of the relative value of SHM is further investigated. The results are provided in Figures 6 and 7. Notice that this case only considers the perfect information, the relative values of SHM are the upper bound of benefits, so these figures indicate that the potential relative benefits of SHM for parallel system are larger than series system. However, the relative values of SHM are not monotonic without a clear tendency with varying correlation, this may be due to the failure correlation is the same as the degradation dependency.

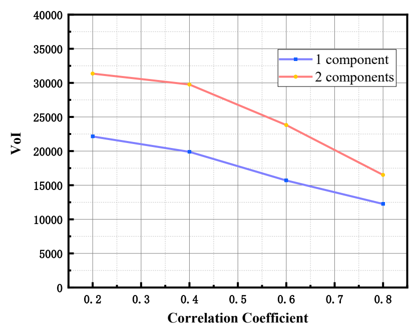

Figure 4: Value of SHM information for the series system with various correlation coefficients.

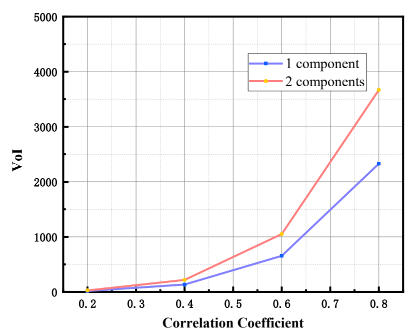

Figure 5: Value of SHM information for the parallel system with various correlation coefficients.

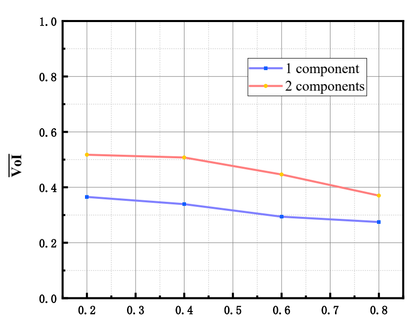

Figure 6: Relative value of SHM information for the series system with various correlation coefficients.

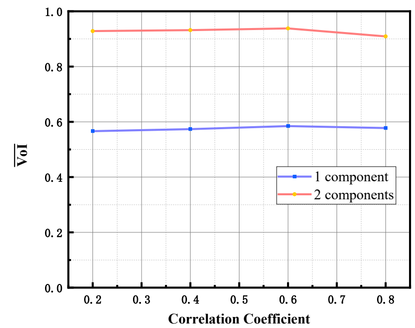

Figure 7: Relative value of SHM information for the parallel system with various correlation coefficients.

7. CONCLUSIONS

This paper aims to illustrate the concept of VoI in the context of life-cycle management of structural systems and to investigate the impact of the existence of dependency of the components on VoI. A generic deterioration function and conditional distribution is used to explain the influence of the number of monitored components on the overall VoI. It is demonstrated how the introduced approach can be applied to determine the optimal number of components to be monitored on the basis of the VoI of the SHM strategy.

It has been found that different system formulations may produce different VoIs from SHM. First, the prior expected life-cycle cost for the series system decreases with the increase of the correlation of component performance whereas that for the parallel system increases. Furthermore, as the correlation is increasing, the value of additional information becomes insignificant gradually, which means that high correlation itself can decrease the uncertainty in decision making. Another finding is that the relative value of information for series systems is lower than that for parallel systems.

An additional finding from the case study results is that the effect of VoI with dependencies from different perspectives cannot be identified clearly, it seems due to the assumption that the component failure and the degradation share the same correlation coefficient, further parametric analysis of correlation coefficient should be made. Also, modelling the systems in an effective computation manner should be studied in the future.

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